Add # 67, 68, 70

Section 6.2 Function/Relations and their Inverses

Name Key

Use the problems from page 396 in your text.

An **INVERSE RELATION is the set of ordered pairs obtained by exchanging the coordinates of each ordered pair.**

What is the inverse of relation in #1 and #2 ?

1. $\{(10,-9), (-3,1), (-5,8)\}$

2. $\{(9,-2), (-1,4), (9,-7), (0,7)\}$

Also, the domain of a relation becomes the range of its inverse, as well, the range of the relation becomes the domain of its inverse.

What is the domain and range of the inverse of #1 and #2 ?

1. D: $\{10, -3, -5\}$ $R = \{-9, 1, 8\}$

2. D: $\{9, -1, \cancel{X}, 0\}$ $R = \{-2, 4, -7, 7\}$

As with relations we can have an inverse of a function. **The inverse of a function f(x) is written as $f(x)^{-1}$.**

Mathematically we find the inverse of a function by

a. Switching x and y of the function and
b. Then solving for y.
c. The resulting equation is the inverse of the original function.

Find the inverse of # 15-19 and #24 ~~#26~~

15. $f(x) = x + 2$

$x = y + 2$

$x - 2 = y$

$f^{-1}(x) = x - 2$

16. $g(x) = 5x$

$g^{-1}(x) \Rightarrow x = 5y$

$\frac{x}{5} = y$

$g^{-1}(x) = \frac{x}{5}$

17. $y = -2x + 1$

$x = -2y + 1$

$x - 1 = -2y$

$\frac{x-1}{-2} = y$

inverse is $y = -\frac{x}{2} + \frac{1}{2}$

19. $y = -\frac{5}{3}x - 8$

$x = -\frac{5}{3}y - 8$

$x + 8 = -\frac{5}{3}y$

$y = -\frac{3}{5}x - \frac{24}{5}$ or $y = \frac{-3x-24}{5}$

24. $h(x) = x^2 + 4$

$x = y^2 + 4$

$x - 4 = y^2$

$\pm\sqrt{x-4} = y$

$h^{-1}(x) = \pm\sqrt{x-4}$

26. $y = (x+1)^2 + 3$

$x = (y+1)^2 + 3$

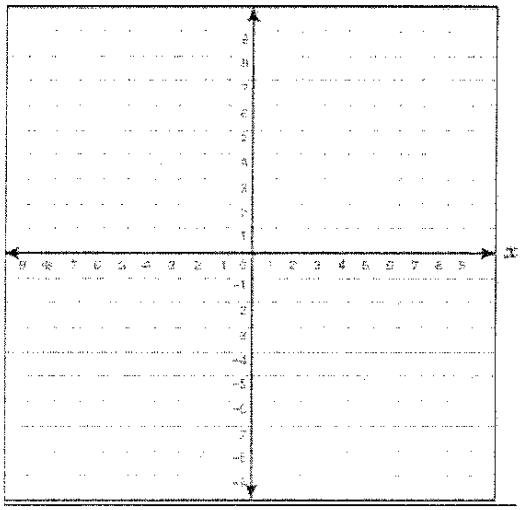
$x - 3 = (y+1)^2$

$-\sqrt{x-3} - 1 = y$

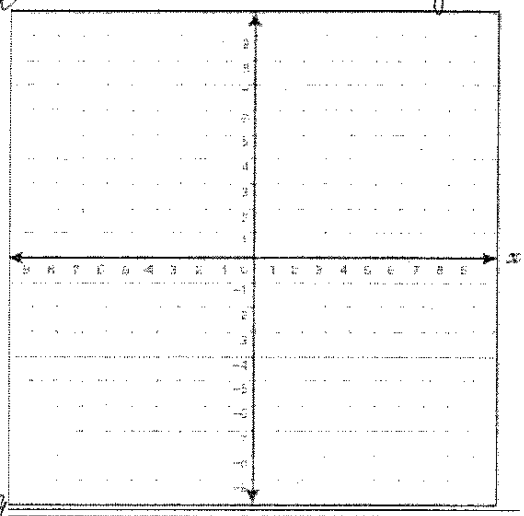
$\sqrt{x-3} - 1 = y$

Now graph the original function f(x) and it inverse g(x), for each of the previous problems on your calculator. Make a quick graph on this sheet for reference. Can you find the line of reflection for each pair of graphs? Line of reflection. $x = 4$

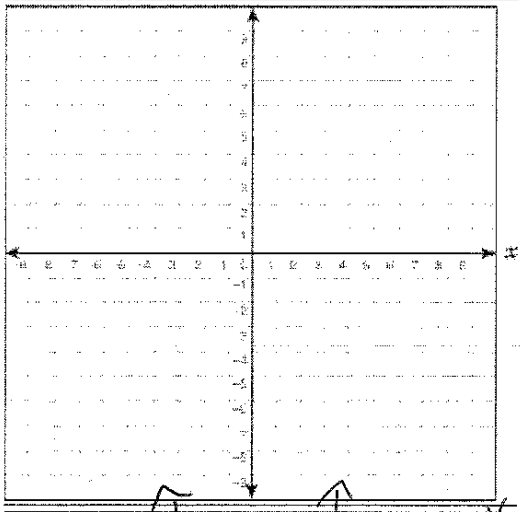
15



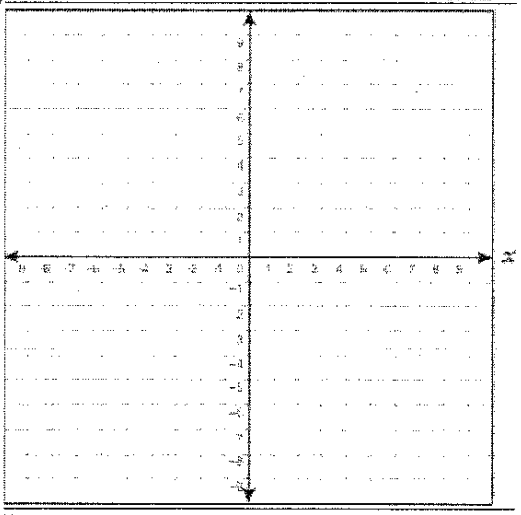
16



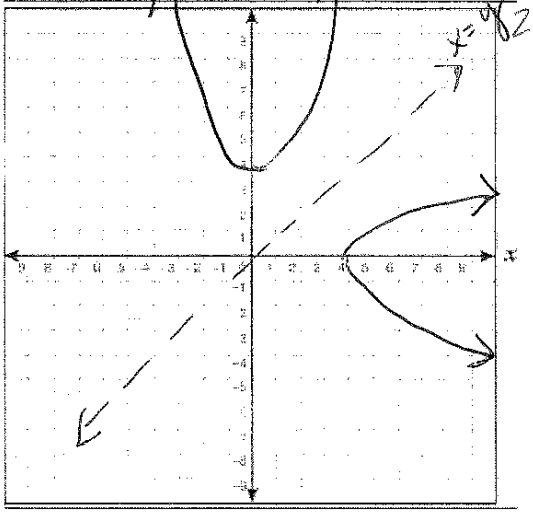
17



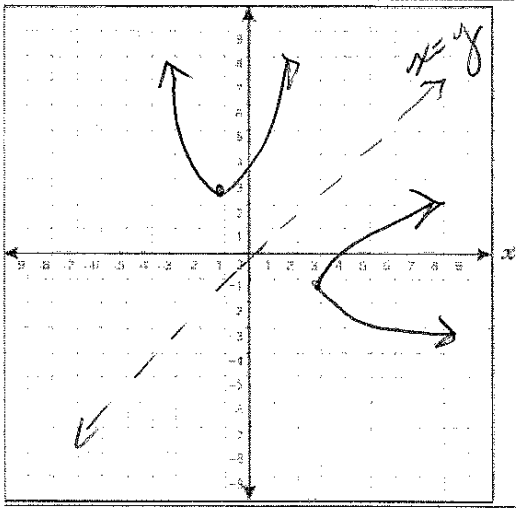
19



24



26



Use Zoom #5 Zoom "square"

Now find $[f \circ g](x)$ and $[g \circ f](x)$ for each function. Where $f(x)$ is function, $g(x)$ is inverse

15. $f(x) = x + 2 \quad f^{-1}(x) = x - 2$ $(f \circ g)(x) = (x-2)+2 = x$ ✓
$(g \circ f)(x) = (x+2)-2 = x$

17. $f(x) = -2x + 1 \quad g(x) = -\frac{x}{2} + \frac{1}{2}$ $(f \circ g)(x) = -2\left(\frac{-x+1}{2}\right)+1 = -(-x+1)+1 = x-1+1 = x$
$(g \circ f)(x) = -\left(\frac{-2x+1}{2}\right)+\frac{1}{2} = -\left(-x+\frac{1}{2}\right)+\frac{1}{2} = x-\frac{1}{2}+\frac{1}{2} = x$

24. $h(x) = x^2 + 4 \quad g(x) = \sqrt{x-4}$ $(h \circ g)(x) = (\sqrt{x-4})^2 + 4 = x - 4 + 4 = x$
$(g \circ h)(x) = \sqrt{x^2+4-4} = \sqrt{x^2} = x$

What conclusion can you make about a function and it's inverse.

The composite of a function and it's inverse is "x".

Two functions f and g are inverses of each other if and only if

$[f \circ g](x) = [g \circ f](x) = x$.

Check to see if #27-35 odds are functions using the above rationale.

27. $f(x) = 2x+3 \quad g(x) = 2x-3$

$2(2x-3)+3 = 4x-6+3$
$\neq x$

Not inverses

29. $f(x) = -\frac{1}{3}x + 3$

$g(x) = -3x + 9$

$[f \circ g](x) = -\frac{1}{3}(-3x+9)+3$
$= x - 3 + 3 = x$

$[g \circ f](x) = -3\left(-\frac{1}{3}x+3\right)+9$
$= x - 9 + 9 = x$

yes

31. $f(x) = \frac{1}{2}x + 5$

$g(x) = 2x - 10$

$[f \circ g](x) = \frac{1}{2}(2x-10)+5$
$= x - 5 + 5 = x$

$g(f(x)) = 2\left(\frac{1}{2}x+5\right)-10$
$= x + 10 - 10 = x$

yes inverses

33. $f(x) = 4x^2$

$g(x) = \frac{1}{2}\sqrt{x}$

$[f \circ g](x) = 4\left(\frac{1}{2}\sqrt{x}\right)^2$
$= 4\left(\frac{1}{4}x\right) = x$

$[g \circ f](x) = \frac{1}{2}\sqrt{4x^2}$
$= \frac{1}{2}(2x) = x$

yes inverses

35. $f(x) = x^2 - 9$

$g(x) = x + 3$

$[f \circ g](x) = (x+3)^2 - 9$
$= x^2 + 6x + 9 - 9$
$= x^2 + 6x$

not inverses

42. No the inverse is not a function

43. yes the inverse is a function

44. The inverse is not a function

48.

x = selling price

$$x + 0.0725x + 350 = 8395.75$$

$$1.0725x = 8045.75$$

$$x = 7501.86$$

$\$7501.86$

49. $C \to F$ is $F(x) = \frac{9}{5}x + 32$

$F^{-1}x = \frac{5}{9}(x - 32)$

$x = \frac{9}{5}y + 32$

$x - 32 = \frac{9}{5}y$

$\frac{5}{9}(x - 32) = y$

$F^{-1}(x)$ converts fahrenheit to celsius

50. 1.852 km is a nautical mile.

$K = 1.852m$

$\frac{K}{1.852} = m$ is the inverse

$$1.852\left(\frac{K}{1.852}\right) = K$$

67. $15 - 6i + 20i - 8i^2$

$15 + 14i + 8$

$23 + 14i$

68. $(\sqrt{6} + i)(\sqrt{6} - i)$

$6 - \sqrt{6}i + \sqrt{6}i - i^2$

$6 + 1$

7

70. $\frac{4 - 3i}{1 + 2i}\cdot\frac{(1 - 2i)}{(1 - 2i)}$

$\frac{4 - 8i - 3i + 6i^2}{1 - 2i + 2i - 4i^2}$

$\frac{-2 - 11i}{1 + 4} = \frac{-2 - 11i}{5}$