Section 6-5: Operations with Radical Expressions

Product Property of Radicals

For any real numbers a and $b \neq 0$ and any integer $n > 1$, if n is even and a and b are both non-negative or if n is odd then,

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient Property of Radicals:

For real numbers a and b, $b \neq 0$ and any integer $n > 1$, if all roots are defined, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

A radical expression is in **simplified form** if:

- the index n is as small as possible
- the radicand contains no factors (other than 1) that are nth powers of an integer or polynomial
- the radicand contains no fractions or decimals
- no radicals appear in a denominator (or a rationalized denominator)

Simplify.

1. $\sqrt{12c^6d^3}$ $\sqrt{4\cdot 3\,c\cdot c\cdot c\cdot c\cdot c\cdot c\cdot d\cdot d\cdot d}$ $2\,c\cdot c\cdot c\cdot d\,\sqrt{3\cdot d}$ $2\lvert c^3\rvert d\,\sqrt{3d}$	2. $\sqrt[3]{27y^{12}z^7} = \sqrt[3]{27\,y^3\cdot y^3\cdot y^3\cdot y^3\,z^3\cdot z^3\cdot z}$ $3y^4z^2\sqrt[3]{z}$ Short cut $3\sqrt[3]{y^{\frac{12}{3}}z^{\frac{7}{3}}} = 3y^4z^2\sqrt{z'}$
3. $\frac{\sqrt{a^9}}{\sqrt{b^5}}\cdot\frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{a^8\cdot a\cdot b}}{\sqrt{b^6}}$ $\frac{a^4\sqrt{ab}}{\lvert b^3\rvert}$	4. $\sqrt[3]{\frac{3}{4y}}\cdot\sqrt{\frac{2y^2}{2y^2}} = \frac{\sqrt[3]{6y^2}}{\sqrt[3]{8y^3}}$ $= \frac{\sqrt[3]{6y^2}}{2y}$
5. $6\sqrt{8c^3d^5}\cdot 4\sqrt{2cd^3}$ $24\sqrt{16c^4d^8}$ $24\cdot 4c^2d^4$ $96c^2d^4$	6. $2\sqrt[4]{3x^3y^2}\cdot 3\sqrt[4]{2x^5y^2}$ $6\sqrt[4]{6x^8y^4}$ $6x^2y\sqrt[4]{6}$

7. $4\sqrt{8}+3\sqrt{50}$ $4\cdot 2\sqrt{2}+3\cdot 5\sqrt{2}$ $8\sqrt{2}+15\sqrt{2}$ $23\sqrt{2}$	8. $5\sqrt{12}+2\sqrt{27}-\sqrt{128}$ $5\sqrt{4\cdot 3}+2\sqrt{9\cdot 3}-\sqrt{64\cdot 2}$ $10\sqrt{3}+6\sqrt{3}-8\sqrt{2}$ $16\sqrt{3}-8\sqrt{2}$

Simplify.

9. $(5+4\sqrt{2})(5-4\sqrt{2})$ $25-20\sqrt{2}+20\sqrt{2}-16\sqrt{4}$ $25-32$ -7	10. $(6\sqrt{3}-5)(2\sqrt{5}+4\sqrt{2})$ $12\sqrt{15}+24\sqrt{6}-10\sqrt{5}-20\sqrt{2}$

You may also need to use conjugates to rationalize a denominator.

11. $\frac{7}{\sqrt{3}+5}\frac{(\sqrt{3}-5)}{(\sqrt{3}-5)}$ $\frac{7\sqrt{3}-35}{\sqrt{9}-5\sqrt{3}+5\sqrt{3}-25}$ $\frac{7\sqrt{3}-35}{-22}$	12. $\frac{4+\sqrt{3}}{\sqrt{5}-\sqrt{2}}\frac{(\sqrt{5}+\sqrt{2})}{(\sqrt{5}+\sqrt{2})}$ $\frac{4\sqrt{5}+4\sqrt{2}+\sqrt{15}+\sqrt{6}}{\sqrt{25}+\sqrt{10}-\sqrt{10}-\sqrt{4}}$ $\frac{4\sqrt{5}+4\sqrt{2}+\sqrt{15}+\sqrt{6}}{3}$