MATHEMATICS • PRACTICE TEST 1 • EXPLANATORY ANSWERS

Question 1. The correct answer is E. To find the total distance, in miles, Kaya ran, you need the sum of $1\frac{2}{5}$ and $2\frac{1}{3}$. To add mixed numbers together, each fraction must have a common denominator. Because 3 and 5 do not have any common factors besides 1, the least common denominator is 3(5), or 15. To convert $\frac{2}{5}$, you multiply by $\frac{3}{3}$. The result is $\frac{6}{15}$. To convert $\frac{1}{3}$, you multiply by $\frac{5}{5}$. The result is $\frac{5}{15}$. To add $1\frac{6}{15}$ and $2\frac{5}{15}$, you first add 1 and 2 and then $\frac{6}{15} + \frac{5}{15}$. The result is $3\frac{6+5}{15}$, or $3\frac{11}{15}$.

B is the most popular incorrect answer and comes from adding the whole number parts and adding the numerators and the denominators separately: $\frac{2+1}{5+3}$. If you chose A, you may have added the whole number parts and multiplied the fractions. If you chose D, you could have incorrectly converted $\frac{2}{5}$ to $\frac{2}{15}$ or $\frac{1}{3}$ to $\frac{1}{15}$ and then added.

Question 2. The correct answer is H. To find an equivalent expression, you can multiply the constants $(3 \cdot 2 \cdot 4)$, combine the x terms $(x^3 x^2 x^2 \Rightarrow x^{3+2+2} \Rightarrow x^7$, because when you have a common base you use the base and add the exponents), and combine the y terms $(y \cdot y \Rightarrow y^1 y^1 \Rightarrow y^{1+1} \Rightarrow y^2)$. The result is $24x^7y^2$.

K is the most common incorrect answer and comes from multiplying the exponents on the x terms instead of adding. If you chose F, you probably added the constants instead of multiplying. If you chose G, you could have added the constants and multiplied the exponents on the x terms instead of adding. If you chose J, possibly you multiplied the exponents on the x terms and y terms instead of adding.

Question 3. The correct answer is A. To find Mr. Dietz's pay per day, you can divide his salary, $22,570, by the number of days he works, 185. His pay per day is $\frac{22,570}{185}$, or $122. When Mr. Dietz takes a day off without pay and the school pays a substitute $80, the school district saves the difference in these amounts, $122 - 80$, or $42.

If you chose B, you probably just picked a number from the problem. If you got E, you probably found Mr. Dietz's pay per day and stopped.

Question 4. The correct answer is J. To find what the student needs to score on the fifth 100-point test to average a score of 80, you need to find the point total for the student so far by adding 65, 73, 81, and 82. That sum is 301. Averaging 80 points on 5 tests means the student must earn 400 points $(80 \cdot 5)$. The score needed on the last test is the difference, $400 - 301$, or 99.

F is the average of the 4 scores, rounded to the nearest whole point. If you chose H, you probably took the average of 65, 73, 81, and 82, averaged that average with 80, and rounded to the nearest whole point. If you chose K, you possibly thought you needed $5(100)$, or 500, points total, and this total is not possible when adding a number 100 or less to 301.

Question 5. The correct answer is D. To find the oxygen saturation loss, you divide the current number of milligrams of dissolved oxygen per liter of water by the dissolved oxygen capacity in milligrams per liter of water, or $\frac{7.3}{9.8}$. Then, you approximate that fraction as a decimal, 0.7449, then convert to a percent, 74.49%, and round to 74%.

If you chose A, you probably divided 9.8 by 7.3, subtracted 1, converted to a percent, and rounded to the nearest whole percent. If you chose B, you probably rounded to the nearest 10%, that is, 74.49% to 70%. If you chose C, you probably just used numbers from the problem.

Question 6. The correct answer is J. To find the length of fence needed to fence a rectangular lot 150 ft by 200 ft, you need to find the perimeter. The formula for the perimeter of a rectangle is 2 times the sum of the length and width, or $P = 2(l + w)$. $2(150 + 200) = 2(350) = 700$.

If you chose G, you probably added the dimensions, but didn't double the sum. If you chose F or H, possibly you used only one dimension and doubled it.

Question 7. The correct answer is A. To find an equivalent expression, multiply a by $b + c - d$. This results in $a(b) + a(c) + a(-d)$, or $ab + ac - ad$.

If you chose E, you probably forgot to distribute the a to c and d.

Question 8. The correct answer is F. To solve for x in the equation $4x + 3 = 9x - 4$, you could subtract $4x$ and add 4 to both sides. That results in the equation $7 = 5x$. Then, dividing both sides by 5, the result is $\frac{7}{5} = x$.

If you chose G, you probably got to $7 = 5x$ and then divided 5 by 7. If you chose H, you probably added $4x$ to $9x$, resulting in $7 = 13x$, and then divided by 13. If you chose J, you might have combined the 3 and −4 and somehow got 1, then got to $1 = 5x$ and divided both sides by 5.

Question 9. The correct answer is C. These 4 numbers will be an arithmetic sequence. In an arithmetic sequence, each pair of successive terms differs by the same amount. To find the difference, you can define d as that difference and let 17 be the first term and 41 the fourth term. By definition, the second term is $17 + d$ and the third term is $(17 + d) + d$. The fourth term, 41, can also be written as $(17 + d + d) + d$. Using that expression you obtain the equation $41 = 17 + d + d + d$, or $41 = 17 + 3d$. After subtracting 17 from both sides, you can then divide by 3, resulting in $8 = d$. The difference is 8. Then, the second term is $17 + 8$, or 25. The third term is $17 + 8 + 8$, or 33.

If you chose A, you probably reasoned that because 41 is the fourth term, the relationship is $4d = 24$ (rather than $3d = 24$) and so the difference is 6. If you chose B, you probably added 7 to the first term and subtracted 7 from the fourth term. If you chose E, you probably added 10 to the first term and subtracted 10 from the fourth term.

Question 10. The correct answer is H. To find what $x^2 + \sqrt{x}$ equals, you need to solve $x^3 = 64$ for x. The solution is $\sqrt[3]{64}$, which is 4. Then, substituting into the original expression, you get $4^2 + \sqrt{4}$. This expression simplifies to $16 + 2$, or 18.

If you chose F, you probably solved $x^3 = 64$ for x and stopped. If you chose G, you could have gotten $x = 4$, used $4(2)$ for 4^2, and added $4(2)$ and 2 to get 10. If you chose J, possibly you got $x = 4$ and then simplified $\sqrt{4}$ to be 4.

Question 11. The correct answer is C. To find the volume, you substitute $\frac{5}{4}$ for r in the equation $V = \frac{4}{3}\pi r^3$. This yields $\frac{4}{3}\pi\left(\frac{5}{4}\right)^3$, or $\frac{125\pi}{48}$. This expression is about 8.18, or 8 to the nearest cubic inch.

If you chose A, you might have substituted to get $\frac{4}{3}\pi\left(\frac{5}{4}\right)$, yielding π, which is about 5. If you chose B, you probably substituted to get $\frac{4}{3}\pi\left(\frac{5}{4}\right)\left(\frac{5}{4}\right)$, yielding $\frac{25}{12}\pi$, or about 7. If you chose D, you probably substituted to get $\frac{4}{3}\pi\left(\frac{5}{4}\right)(3)$, yielding 5π, or about 16.

Question 12. The correct answer is K. The probability that the marble chosen will not be white when 8 marbles are red, 6 are blue, and 6 are white is the number of favorable outcomes divided by the total number of possible outcomes. The number of *favorable* outcomes is 14 because there are 8 red marbles and 6 blue marbles—a total of 14 marbles. The total number of *possible* outcomes is $8 + 6 + 6 = 20$, the total number of marbles. Thus, the probability of the marble NOT being white is $\frac{8+6}{8+6+6} = \frac{14}{20} = \frac{7}{10}$.

If you chose G, you probably added the number of blue marbles and the number of white marbles and divided by the total number of marbles: $\frac{6+6}{20} = \frac{12}{20} = \frac{3}{5}$. If you chose H, you probably found $\frac{8+8}{8+6+6} = \frac{16}{20} = \frac{4}{5}$. If you chose J, you probably found the probability of choosing a white marble: $\frac{6}{8+6+6} = \frac{6}{20} = \frac{3}{10}$.

Question 13. The correct answer is D. To find the number of sports awards earned, the number of participants in each sport is multiplied by the ratio for that sport and then the 4 products are added. This is a matrix multiplication.

$$[40 \ 60 \ 80 \ 80] \begin{bmatrix} 0.3 \\ 0.4 \\ 0.2 \\ 0.5 \end{bmatrix} = 40(0.3) + 60(0.4) + 80(0.2) + 80(0.5) = 12 + 24 + 16 + 40 = 92$$

If you chose B, you probably reversed the order on the first matrix to get $80(0.3) + 80(0.4) + 60(0.2) + 40(0.5) = 24 + 32 + 12 + 20 = 88$. If you chose C, you probably totaled the number of athletes and multiplied it by the average of the ratios, $260(0.35)$, which is 91.

Question 14. The correct answer is H. To find the average number of students enrolled per section of Algebra I, you add up the students in all the sections and divide by the number of sections. Thus, you add $24 + 25 + 29$ and get 78, then divide by 3. This results in an average of 26 students enrolled per section in Algebra I.

If you chose G, you could have found the median (or middle number) of 24, 25, and 29. Sometimes, *average* can mean the median or the mode. For this test, the directions say that, unless otherwise stated, "The word *average* indicates arithmetic mean." If you chose J, you likely found the average of 25 and 29.

Question 15. The correct answer is C. The total number of calculators available is $30 - 2 + 30 - 6 = 52$. To find the class periods for which there are not enough school calculators, find the total needed for each period, as given in the table below.

Period	1	2	3	4	6
Calculators needed	21	46	48	57	19

The only entry in the table more than 52 is 57 for Period 4.

If you chose D, possibly you looked at the Algebra I rows in the table and saw that Section B and Section C could not both be covered by the available calculators, and these sections are in Period 3 and Period 4. If you chose E, you probably used 60 for the available number of calculators and did not take into account the 8 calculators that are being repaired and are unavailable.

Question 16. The correct answer is **H.** Because the sum of each row is equivalent, the sum of Row 1 is the same as the sum of Row 2.

$$\text{Row 1:} \qquad x + 8x + (-3x) \Rightarrow 6x$$
$$\text{Row 2:} \qquad -2x + \; ? \; + \; 6x \Rightarrow 4x + \; ?$$

The question mark must represent $2x$. You could have done this with other rows, columns, or diagonals.

If you chose G, you probably just added the first and last entries in either Row 2, Column 2, or one of the diagonals. If you chose K, you may have thought that each sum must be 0 and found that $-4x$ would make the sums of Row 2, of Column 2, and of both diagonals be 0.

Question 17. The correct answer is **E.** The x-coordinate is positive if A is to the right of the y-axis. The y-coordinate is positive if A is above the x-axis. The table below shows the sign of x and the sign of y in the four quadrants.

	Sign of:	
Quadrant	x	y
I	+	+
II	−	+
III	−	−
IV	+	−

Thus, the signs are opposite in Quadrants II and IV only.

If you chose C or D, you probably got confused about where x and y are positive and negative or about the order of the quadrants.

Question 18. The correct answer is J. To find the number of distinct outfits that Kareem can select from 4 sweaters, 6 shirts, and 3 pairs of slacks, multiply the numbers of the 3 different clothing pieces together. Thus, there are 4(6)(3), or 72, distinct outfits that Kareem can select. The figure below shows that for each sweater, there are 6 shirts, and for each shirt, there are 3 pairs of slacks.
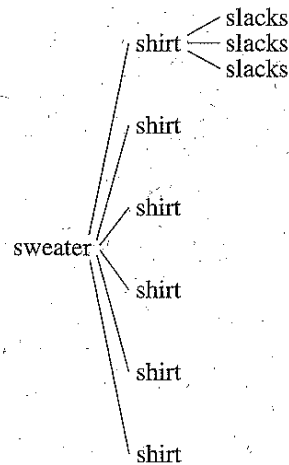
If you chose F, you probably added the 3 numbers together, getting 4 + 6 + 3 = 13.

Question 19. The correct answer is A. To find the number of tons of sand needed to produce 3,000 barrels of a tarry material that requires 100,000 tons of sand for 60,000 barrels, you can set up a proportion with ratios of tons of sand to barrels of tarry material, such as

$\frac{100,000}{60,000} = \frac{\text{tons of sand}}{3,000}$, which results in 5,000 tons of sand.

If you chose B, you probably calculated $\frac{60,000\,(30,000)}{100,000}$.

Question 20. The correct answer is H. The figure below shows the rectangle and a diagonal. To find the length of the diagonal, you could use the Pythagorean theorem because the sides of the rectangle are the legs of a right triangle and the diagonal of the rectangle is the hypotenuse of the right triangle. Then $h^2 = 72^2 + 54^2 \Rightarrow h = 90$.
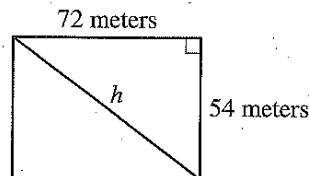
G is the average of 54 and 72. If you chose J, you probably added 54 and 72.

Question 21. The correct answer is A. To find an equivalent expression for $\frac{x}{y}$, you must either multiply or divide both the numerator and the denominator by the same value.

Multiplying $\frac{x}{y}$ by $\frac{z}{z}$ yields $\frac{x \cdot z}{y \cdot z}$.

If you chose B, you probably thought you could multiply by the expression and obtain an equivalent expression, but if $\frac{x}{y} = \frac{2}{3}$, then $\frac{x^2}{y^2} = \frac{4}{9} \neq \frac{2}{3}$. If you chose C, you probably thought you could

multiply by the reciprocal and obtain an equivalent expression, but if $\frac{x}{y} = \frac{2}{3}$, then

$\frac{x \cdot y}{y \cdot x} = \frac{2 \cdot 3}{3 \cdot 2} = 1 \neq \frac{2}{3}$. If you chose E, you probably thought you could add the same number to both

the numerator and the denominator and obtain an equivalent expression, but if $\frac{x}{y} = \frac{2}{3}$ and $z = 2$,

then $\frac{x+2}{y+2} = \frac{2+2}{3+2} = \frac{4}{5} \neq \frac{2}{3}$.

Question 22. The correct answer is H. To find the slope-intercept form of the equation $8x - y - 6 = 0$, you could first add 6 and subtract $8x$ from both sides of the equation to get $-y = -8x + 6$. Then, multiply by -1 to get $y = 8x - 6$.

If you chose F, you probably forgot to switch the sign on $8x$ when you multiplied by -1. If you chose G, you probably just dropped the sign on $-y$. If you chose J, you probably forgot to multiply 6 by -1 in the last step.

Question 23. The correct answer is B. To solve the quadratic equation $x^2 - 36x = 0$ for x, you could factor the left side to $x(x - 36) = 0$ and apply the zero product rule. Thus, $x = 0$ or $x - 36 = 0$, which implies $x = 0$ or $x = 36$. The solution given as an answer choice is 36.

If you chose C, you probably divided 36 by 2. If you chose D, you probably dropped the x in the second term and solved $x^2 = 36$ for a positive value. If you chose E, you probably dropped the x in the second term and solved $x^2 = 36$ for negative value because there was a negative sign in the original equation.

Question 24. The correct answer is G. To find $\tan R$ in $\triangle RST$, take the ratio of the length of the opposite leg to the length of the adjacent leg, or ST to RS, or r to t, or $\frac{r}{t}$.

F is $\sin R$, H is $\cot R$, J is $\cos R$, and K is $\sec R$. If you did not get the correct answer, it would be wise to review trigonometric ratios in a right triangle.

Question 25. The correct answer is D. To find the radius, you can use the right triangle shown on the diagram. Half the length of the chord is 12 inches, which is the length of one leg. The other leg is 5 inches long, and the hypotenuse is r inches long. (This is a right triangle because the distance between a point and a line must be measured perpendicular to the line.) Using the Pythagorean theorem $r^2 = 12^2 + 5^2 \Rightarrow r^2 = 169 \Rightarrow r = 13$ inches.

A is $24 + 5$, which is clearly much longer than the radius. If you chose B, you probably used 24 and 5 for the leg lengths and got $r = \sqrt{601}$, which is about $r = 24.5$ inches. Choice C is closest to $5 + 12$. Going along the radius line must be shorter than going along the 2 legs of the triangle.

Question 26. The correct answer is H. To find the force F (in newtons) corresponding to a spring length, L, of 0.18 meters when the relationship is given by the equation $L = \frac{2}{3}F + 0.03$, you would substitute 0.18 for L to get $0.18 = \frac{2}{3}F + 0.03$. After subtracting 0.03 from both sides, you'd get $0.15 = \frac{2}{3}F$. Then, after multiplying by $\frac{3}{2}$, you'd get $0.225 = F$.

G is the result of replacing F by 0.18 and solving for L. If you chose J, possibly you got $0.225 = F$ and added 0.03.

Question 27. The correct answer is B. To find the uniform depth the 10,000 cubic yards of snow would be on the rectangular football field with dimensions 120 yards by 53.5 yards, you would substitute in the formula for volume, V, of a rectangular prism with the height h, length l, and width w, which is $V = lwh$. After substituting you should have $10,000 = 120(53.5)(h)$, or $10,000 = 6,420h$. Thus, $h = \frac{10,000}{6,420}$, or about 1.558. And 1.558 is between 1 and 2.

If you chose A, you probably took $\frac{6,420}{10,000}$ and got 0.642, which is less than 1. If you chose C or D, you probably used the wrong dimensions or made a mistake in calculations.

Question 28. The correct answer is G. To find the length of $\overline{QR}$ in $\triangle PQR$, where $\overline{PR}$ is 16 feet long and $\sin \angle P = \frac{3}{5}$, use the definition of sine: the ratio of the length of the opposite side to the length of the hypotenuse. In $\triangle PQR$, $\sin \angle P = \frac{QR}{PR}$. After substituting for $\sin \angle P$ and PR, the length of the hypotenuse, you obtain $\frac{3}{5} = \frac{QR}{16} \Rightarrow 5 \cdot QR = 48 \Rightarrow QR = 9.6$ feet.

F is $\frac{1}{2}$ of PR. If you chose H, you probably found $\cos \angle P = 0.8$ and then multiplied 16(0.8) to get 12.4.

Question 29. The correct answer is B. To find the fraction of cars assembled in Coupeville, you would divide the number assembled in Coupeville by the total number assembled. The table below shows the conversion of car symbols to numbers for the 4 cities and the total.

City	Number of cars assembled
Car Town	40,000
Coupeville	25,000
Truck City	20,000
Sedan Falls	15,000
All	100,000

The fraction assembled in Coupeville is $\frac{25,000}{100,000}$, or $\frac{1}{4}$.

If you chose A, you probably found the fraction for Truck City, $\frac{20,000}{100,000}$, or $\frac{1}{5}$. If you chose C, you may have thought a half car represented 10,000, so your fraction was $\frac{30,000}{110,000}$, or $\frac{3}{11}$. If you chose D, you probably used the fraction $\frac{30,000}{100,000}$, or $\frac{3}{10}$. If you chose E, you probably used the number in Coupeville divided by the total number from the other 3 cities, $\frac{25,000}{75,000}$, or $\frac{1}{3}$.