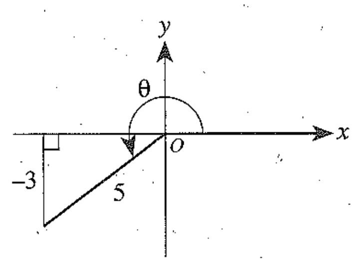
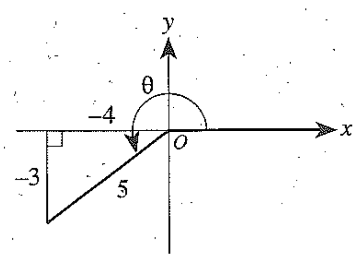
**Question 54. The correct answer is J.** One way to find tan θ, given that $\sin\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, is to first find cos θ, then find $\frac{\sin\theta}{\cos\theta}$ (which is equivalent to tan θ). To find cos θ, use the facts that $\cos\theta < 0$ in Quadrant III and that $\sin^2\theta + \cos^2\theta = 1$. Substituting, you get $\left(-\frac{3}{5}\right)^2 + \cos^2\theta = 1$, or $\frac{9}{25} + \cos^2\theta = 1$. After subtracting $\frac{9}{25}$, you get $\cos^2\theta = \frac{16}{25}$. After taking the square root of both sides, you get $\cos^2\theta = \pm\frac{4}{5}$. Because $\cos\theta < 0$, $\cos\theta = -\frac{4}{5}$. Substituting into $\frac{\sin\theta}{\cos\theta}$ gives you $\frac{-\frac{3}{5}}{-\frac{4}{5}}$, which simplifies to $\frac{3}{4}$.

Another way you could do this problem is to construct an angle in Quadrant III with $\sin\theta = -\frac{3}{5}$. (Recall that sine is the ratio of opposite to hypotenuse.) Such an angle is shown below.



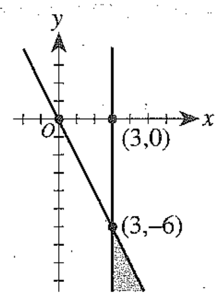

By the Pythagorean theorem, the missing side of the right triangle is 4 coordinate units long, and the directed distance along the side is –4. The figure below shows this.




From this right triangle, knowing that tangent is $\frac{\text{opposite}}{\text{adjacent}}$, you can get $\tan\theta = \frac{-3}{-4} = \frac{3}{4}$.

G comes from using $\frac{4}{5}$ for cos θ instead of $-\frac{4}{5}$. If you chose H, you might have mixed up the definition of sine or tangent in the right triangle.

**Question 55. The correct answer is A.** To find the system of inequalities represented by the shaded region of the graph below,

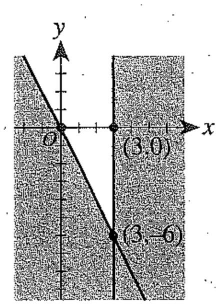



you could first find the equations of the line through (0,0) and (3,–6) and the line through (3,0) and (3,–6). Those are $y = -2x$ and $x = 3$. It is clear from the graph that the inequality that represents the shaded side of $x = 3$ is $x \geq 3$. For the other line, if you test (3,0), you find it satisfies $y > -2x$. Because (3,0) is on the wrong side (the unshaded side) of $y = -2x$, the correct inequality is $y \leq -2x$.

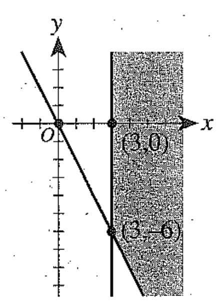
The graphs of the incorrect answer choices are shown below.

Choice C is the most popular incorrect answer (about as many people choose this as choose the correct answer). The inequality sign is backwards for the line $y = -2x$.

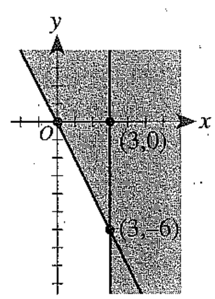
Choice B differs from the correct answer only in the "or" connector. The graph of B includes points that satisfy one of the inequalities but not necessarily the other inequality, while the "and" connector means the graph can only include points that satisfy both inequalities.



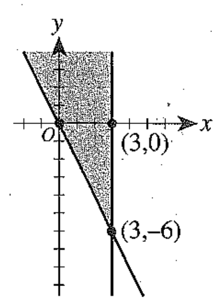
Graph of B



Graph of C



Graph of D



Graph of E

## MATHEMATICS • PRACTICE TEST 1 • EXPLANATORY ANSWERS

**Question 56. The correct answer is K.** To find $f(x + h)$ when $f(x) = x^2 - 2$, you would substitute $(x + h)$ for $x$ in $f(x) = x^2 - 2$. The result is $(x + h)^2 - 2$. Multiplying out $(x + h)^2$ yields $x^2 + xh + xh + h^2$, or $x^2 + 2xh + h^2$. Then add $-2$ to the result.

If you chose G, you interpreted $f(x + h)$ as $f(x) + h$. If you chose H, you replaced $(x + h)^2$ with $x^2 + h^2$. If you chose J, you found $(x + h)^2$.

**Question 57. The correct answer is A.** It might be surprising to see that the graph of this complicated function looks almost like a line. The equation $y = \frac{2x^2+x}{x}$ can be written as $y = \frac{x(2x+1)}{x}$. This is equivalent to $y = 2x + 1$ except when $x = 0$. When $x = 0$, the original equation is undefined. So the correct graph is $y = 2x + 1$ with a point removed where $x = 0$.

If you noticed that the function was undefined when $x = 0$, you may have thought the open dot belonged at (0,0). That leaves B as the only answer choice that also goes through (1,3).

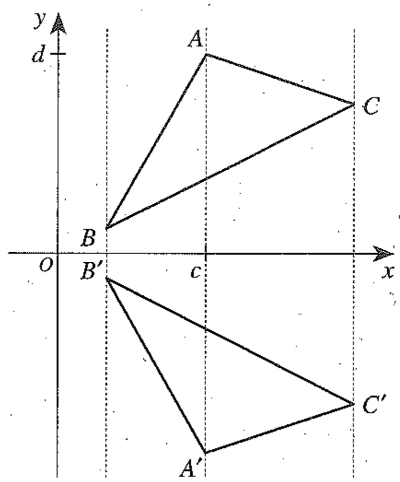
Choice C is the only one that involves (0,2), and you may have gotten this by substituting $x = 0$ to get $y = \frac{2(0^2)+0}{0}$, and decided all the zeros could be dropped to yield $y = 2$.

If you chose D, you may have "cancelled" $x$'s as $y = \frac{2(x^2)+\cancel{x}}{\cancel{x}}$ to get $y = 2x^2 + 1$. You could have eliminated this answer by testing (−1,3) in the original equation, but testing (1,3) would not have been enough.

Choice E can come from "cancelling" $x$'s as $y = \frac{2x^2+\cancel{x}}{\cancel{x}}$ to get $y = 2x^2$. You could have eliminated this answer by testing (1,2) in the original equation.

## MATHEMATICS • PRACTICE TEST 1 • EXPLANATORY ANSWERS

**Question 58. The correct answer is F.** To find the coordinates of vertex $A$ after it is reflected across the $x$-axis, notice that a reflection across the $x$-axis does not change the $x$-coordinate but does change the sign of the $y$-coordinate. You might sketch or imagine a figure like the one below. Thus, the reflection of $A(c,d)$ across the $x$-axis is $A'(c,-d)$.




G gives $A$ reflected across the $y$-axis. H gives $A$ reflected across (0,0). J gives $A$ reflected over the line $y = x$ and is the most popular answer.

**Question 59. The correct answer is A.** To obtain an expression for $y$ in terms of $x$ when $x = 2t - 9$ and $y = 5 - t$, you can first solve $x = 2t - 9$ for $t$ by adding 9 to both sides to get $x + 9 = 2t$. Then, divide both sides by 2 to get $\frac{x+9}{2} = t$. Substitute that expression for $t$ into $y = 5 - t$ to get $y = 5 - \frac{x+9}{2}$. To simplify the right side, rewrite 5 as $\frac{10}{2}$ and then combine the 2 fractions together to get $y = \frac{10-(x+9)}{2}$. You can then distribute and combine like terms to get $y = \frac{1-x}{2}$.

If you chose B, you probably got $y = \frac{10-(x+9)}{2}$ and simplified it to $y = \frac{19-x}{2}$. If you chose C, you may have substituted $2x - 9$ for $t$ in $y = 5 - t$, which results in $y = 5 - (2x - 9)$. After distributing, this would be $y = 5 - 2x + 9$, or $y = 14 - 2x$.

## MATHEMATICS • PRACTICE TEST 1 • EXPLANATORY ANSWERS

**Question 60.** The correct answer is K. To find $\sin \frac{\pi}{12}$ using $\sin(\alpha - \beta) = (\sin \alpha)(\cos \beta) - (\cos \alpha)(\sin \beta)$ given that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, you can first substitute $\frac{\pi}{3}$ for $\alpha$ and $\frac{\pi}{4}$ for $\beta$ and get $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \left(\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$. Using the table of values to substitute in that equation, you get $\sin \frac{\pi}{12} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$.

H comes from calculating $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$, which is $\left(\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{3}\right) - \left(\sin\frac{\pi}{4}\right)\left(\cos\frac{\pi}{4}\right)$. If you chose J, you probably just used $\sin\frac{\pi}{3} - \sin\frac{\pi}{4}$.

**2** △ △ △ △ △ △ △ △ △ **2**

60. What is $\sin \frac{\pi}{12}$ given that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ and that $\sin(\alpha - \beta) = (\sin \alpha)(\cos \beta) - (\cos \alpha)(\sin \beta)$ ?

(Note: You may use the following table of values.)

| $\theta$ | $\sin\theta$ | $\cos\theta$ |
|---|---|---|
| $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |
| $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |

F. $\frac{1}{4}$

G. $\frac{1}{2}$

H. $\frac{\sqrt{3}-2}{4}$

J. $\frac{\sqrt{3}-\sqrt{2}}{2}$

K. $\frac{\sqrt{6}-\sqrt{2}}{4}$

**DO YOUR FIGURING HERE.**

**END OF TEST 2**

**STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.**

**DO NOT RETURN TO THE PREVIOUS TEST.**

60. What is $\sin\frac{\pi}{12}$ given that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ and that $\sin(\alpha - \beta) = (\sin\alpha)(\cos\beta) - (\cos\alpha)(\sin\beta)$ ?

(Note: You may use the following table of values.)

| $\theta$ | $\sin\theta$ | $\cos\theta$ |
|---|---|---|
| $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |
| $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |

F. $\frac{1}{4}$

G. $\frac{1}{2}$

H. $\frac{\sqrt{3}-2}{4}$

J. $\frac{\sqrt{3}-\sqrt{2}}{2}$

K. $\frac{\sqrt{6}-\sqrt{2}}{4}$

**DO YOUR FIGURING HERE.**

**END OF TEST 2**

**STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.**

**DO NOT RETURN TO THE PREVIOUS TEST.**

## MATHEMATICS • PRACTICE TEST 1 • EXPLANATORY ANSWERS

**Question 60.** The correct answer is K. To find sin $\frac{\pi}{12}$ using $\sin(\alpha - \beta) = (\sin \alpha)(\cos \beta) - (\cos \alpha)(\sin \beta)$ given that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$, you can first substitute $\frac{\pi}{3}$ for $\alpha$ and $\frac{\pi}{4}$ for $\beta$ and get $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \left(\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$. Using the table of values to substitute in that equation, you get $\sin\frac{\pi}{12} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$.

H comes from calculating $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$, which is $\left(\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{3}\right) - \left(\sin\frac{\pi}{4}\right)\left(\cos\frac{\pi}{4}\right)$. If you chose J, you probably just used $\sin\frac{\pi}{3} - \sin\frac{\pi}{4}$.