Section 6-1 Operations on Functions

1. Given $f(x) = x^2 + 5x - 2$ and $g(x) = 3x - 2$, find each function. Indicate any restrictions in the domain or range.

$(f+g)(x) = f(x) + g(x) = (x^2+5x-2) + (3x-2) = x^2+8x-4$

$(f-g)(x) = f(x) - g(x) = (x^2+5x-2) - (3x-2) = x^2+2x$

$(f \cdot g)(x) = f(x) \cdot g(x) = (x^2+5x-2)(3x-2) = 3x^3 - 2x^2 + 15x^2 - 10x - 6x + 4 = 3x^3+13x^2-16x+4$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+5x-2}{3x-2};\ x \neq \frac{2}{3}$

the denominator cannot be zero... $3x-2 \neq 0$, $3x \neq 2$, $x \neq \frac{2}{3}$

A **composition (composite) function** is $f(g(x))$ or $[f \circ g](x)$, read *f* of *g* of *x*.

2. Given $f(x) = \{(3,-2), (-1,-5), (4,7), (10,8)\}$ and $g(x) = \{(4,3), (2,-1), (9,4), (3,10)\}$, find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each composed function.

$[f \circ g](x) = f(g(x))$

$= \{(4,-2), (2,-5), (9,7), (3,8)\}$

$D = \{2, 3, 4, 9\}$

$R = \{-5, -2, 7, 8\}$

$[g \circ f](x) = g(f(x))$

$= \{(3, \ ), (-1, \ ), (4, \ ), (10, \ )\}$

$= \emptyset$

since $f(x)$ $(3,-2)$ there is no -2 in the domain of $g(x)$

3. Given $f(x) = x^2 + 2$ and $g(x) = x - 6$, find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each composed function.

$[f \circ g](x) = f(g(x)) = f(x-6) = (x-6)^2 + 2 = x^2 - 12x + 36 + 2 = x^2 - 12x + 38$

$D = \{x \mid x = \mathbb{R}\}$

$R = \{y \mid y \geq 2\}$

$[g \circ f](x) = g(f(x)) = g(x^2+2) = x^2 + 2 - 6 = x^2 - 4$

$D = \{x \mid x = \mathbb{R}\}$

$R = \{y \mid y \geq -4\}$

4. The Music Box offers both an in-store \$35 rebate and a 15% discount on a digital audio player that normally sells for \$300. Which provides the better price: taking the discount before the rebate or taking the rebate before the discount?

Rebate: $f(x) = x - 35$

15% Disc: $g(x) = 0.85x$

Rebate first: $g(f(x)) = g(x-35) = 0.85(x-35) = 0.85x - 29.75$

15% Disc first: $f(g(x)) = f(0.85x) = 0.85x - 35$

$g(f(300)) = 0.85(300) - 29.75 = \$225.25$

$f(g(300)) = 0.85(300) - 35 = \$220$

taking the discount first provides the better price (you save \$5.25)